Spin-squeezed Ground States in the Bilayer Quantum Hall Ferromagnet

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(November 5, 1997)

A "squeezed-vacuum" state considered in quantum optics is shown to be realized in the ground-state wavefunction for the bilayer quantum Hall system at the total Landau level filling of $\nu = 1/m$ (m: odd integer). This is derived in the boson approximation, where a particle-hole pair creation across the symmetric-antisymmetric gap, $\Delta_{\rm SAS}$, is regarded as a boson. In terms of the pseudospin describing the layers, the state is a spin-squeezed state, where the degree of squeezing is controlled by the layer separation and $\Delta_{\rm SAS}$. An exciton condensation, which amounts to a rotated spin-squeezed state, has a higher energy due to the degraded SU(2) symmetry for $\Delta_{\rm SAS} \neq 0$.

To regard a two-level system as a pseudospin has a long history. The pseudospin ferromagnetism in the bilayer quantum Hall (QH) system [1] is an outstanding recent example for a bulk system. Although the introduction of the pseudospin starts from a simple definition of assigning the upper/lower levels (or layers) to the pseudospin ↑ / ↓, the "Pauli matrices" for them have a Lie-group structure, long known in the molecular physics [2], so that we have an interesting problem of how quantum fluctuations dominate the physics in the bulk.

Here we wish to point out that a boson "squeezed vacuum" state considered in quantum optics is shown to be realized in the ground-state wavefunction for the bilayer QH system at the total Landau level filling of $\nu = 1/m$ (m: odd integer). *Squeezing*, in general, is a redistribution of quantum fluctuations between two non-commuting observables with a preserved minimum uncertainty product. Here the boson refers to a particle-hole pair created across the symmetric-antisymmetric gap due to the interlayer electron tunneling, and we can start from the vacuum of the boson. In terms of the pseudospin describing the layers, we can also call the state a "spin-squeezed" state. The squeezing is caused by the electron correlation in the bilayer QH system, and the degree of squeezing is controlled by two factors governing the degradation of the pseudospin rotational symmetry, the layer separation and $\Delta_{\rm SAS}$.

To be more precise, the degraded rotational symmetry results from the difference between the intralayer and interlayer Coulomb interaction for a finite layer separation d, and from the interlayer tunneling [3]. Since the two layers tend to have equal numbers of electrons due to a capacitive charging energy, the z component of the total pseudospin (half the difference in the numbers) tends to vanish, $\langle s_z \rangle = 0$, in the ground state. Thus the pseudospin will lie in the xy plane, where the system maintains an invariance under rotations about the z axis, *i.e.*, the SU(2) symmetry is reduced to U(1) [4].

The electron correlation alone pushes the ground state for the total Landau level filling $\nu = 1/m$ (m: odd integers) towards a ferromagnetic one, *i.e.*, Halperin's Ψ_{mmm} [5], so the bilayer $\nu = 1/m$ QH system behaves like an easy-plane XY itinerant-electron ferromagnet. The U(1) symmetry is further degraded in the presence of the interlayer tunneling, since the tunneling amplitude behaves like a magnetic field acting on the pseudospin, as seen in the tunneling Hamiltonian, $H_{\rm T} = -\Delta_{\rm SAS}\, s_x$. This favors another pseudospin-polarized state $\Psi_{\rm sym}$ for $\nu = 1$ by pushing the electrons into the symmetric band [3].

So a bilayer QH system is characterized by two dimensionless parameters, d/ℓ and $\Delta_{\rm SAS}/(e^2/\epsilon\ell)$ (ℓ : the magnetic length, ϵ: dielectric constant). The ground state for $\nu = 1$ is considered to evolve continuously from tunneling-dominated (single-particle like) to correlation-dominated (many-body) as $\Delta_{\rm SAS}$ is decreased, in agreement with experimental results for $\nu = 1$ [6]. The fact that there is no intervening non-ferromagnetic region between the two regimes has been confirmed in an exact numerical calculation [7].

Thus the energetics in the bilayer QH system has been elaborated, which includes the random-phase approximation (RPA) [8], the boson approximation [3,9], macroscopic field-theoretical approaches [4]. However, the ground-state *wavefunction* has been discussed only in the Hartree-Fock (HF) approximation [10], which cannot fully reflect the degraded symmetry of the system, and a clear picture of wavefunctions has yet to come. That is exactly the purpose of the present paper, where the squeezing comes in to give the ground-state wavefunction as a function of d and $\Delta_{\rm SAS}$. The exciton condensation, another long-standing problem in this system, may be described in the present formalism as a "rotated spin-squeezed" state, which is shown to have a higher energy due to the symmetry degraded from SU(2) for $\Delta_{\rm SAS} \neq 0$.

We start from the effective Hamiltonian for the $\nu = 1$ pseudospin ferromagnet [3,9]. In order to exploit the full rotational symmetry, we use a spherical system, *i.e.*, an N-electron system on a sphere whose surface is passed through by $2S$ flux quanta, where $2S = N - 1$ for $\nu = 1$. The Hamiltonian is given by

$$\mathcal{H}_{\rm eff} = \sum_{L,M} \Bigg[(e_L + \lambda_L)\, C^\dagger_{LM}\, C_{LM} + \frac{\lambda_L}{2} \Big(C^\dagger_{LM}\, C^\dagger_{L,-M} + C_{LM}\, C_{L,-M} \Big) \Bigg], \qquad (1)$$

$$e_L \equiv \Delta_{\rm SAS} + \sum_{J=0}^{2S} (2J+1)\,(-1)^{2S-J}\,V_J^{\uparrow\downarrow} \times \left[\frac{1}{2S+1} - (-1)^{2S-J} \left\{ \begin{array}{ccc} S & S & L \\ S & S & J \end{array} \right\} \right], \qquad (2)$$

$$\lambda_L \equiv \sum_{J}^{2S-J:\,{\rm odd}} (2J+1)\,(V_J^{\uparrow\uparrow} - V_J^{\uparrow\downarrow})\,(-1)^{2S-J} \times \left\{ \begin{array}{ccc} S & S & L \\ S & S & J \end{array} \right\}. \qquad (3)$$

The interaction matrix elements can be expressed in terms of Wigner's $6j$ symbol $\{{}^{SSL}_{SSJ}\}$ [9], while $C^\dagger_{LM} \equiv \sum_{j,k} \langle Sj; S, -k | LM\rangle\, a^\dagger_{j\,{\rm a}}\, (-1)^{S-k}\, a_{k\,{\rm s}}$ creates an exciton (a hole in the symmetric band and a particle in the antisymmetric one) with the total angular momentum L and its z component M, where $\langle Sj; S, -k | LM\rangle$ is the Clebsch-Gordan coefficient, and $a^\dagger_{j\,{\rm a}}$ ($a^\dagger_{k\,{\rm s}}$) is a creation operator for an antisymmetric (symmetric) state with the Landau orbit j (k).

The inter-particle interaction is projected onto the components $\{V_J^{\sigma\sigma'}\}$ (the Haldane pseudopotential) for the relative angular momentum $2S-J$, where $V_J^{\uparrow\uparrow} = V_J^{\downarrow\downarrow}$ and $V_J^{\uparrow\downarrow} = V_J^{\downarrow\uparrow}$ are the intra- and inter-layer pseudopotentials, respectively. The terms in the form of $C^\dagger C$ has a coefficient of $e_L + \lambda_L$ (involving $\Delta_{\rm SAS}$ and particle-hole correlation energies), while we have off-diagonal terms (in the form of $C^\dagger C^\dagger$ or CC) that create or annihilate excitons in pairs, whose coefficient arises from the difference between the intralayer and interlayer interactions.

In order to obtain the ground-state wavefunction, we can diagonalize the Hamiltonian, eq.(1). Namely, we can look at how the ground state, which is $|\Psi_{\rm sym}\rangle \equiv \prod_j a^\dagger_{j\,{\rm s}}\,|0\rangle$ in the limit of large $\Delta_{\rm SAS}$, evolves. The starting state $\Psi_{\rm sym}$ satisfies the relation $C_{LM}\,|\Psi_{\rm sym}\rangle = 0$ due to the definition of C_{LM}. Since we have $[\,C_{LM}, C_{L'M'}\,] = [\,C^\dagger_{LM}, C^\dagger_{L'M'}\,] = 0$, we can call $\Psi_{\rm sym}$ the vacuum state of the *boson* $\{C_{LM}\}$ if the further commutation relation $[\,C_{LM}, C^\dagger_{L'M'}\,] = \delta_{L\,L'}\,\delta_{M\,M'}$ is satisfied. We have in fact

$$\left[\, C_{LM}, C^\dagger_{L'M'} \,\right] \;\simeq\; \delta_{L\,L'}\,\delta_{M\,M'} \left(1 - \frac{N_{\rm h}}{2S+1} - \frac{N_{\rm p}}{2S+1} \right), \qquad (4)$$

where we have substituted $a_{j\,{\rm s}}\,a^\dagger_{k\,{\rm s}}$ and $a^\dagger_{j\,{\rm a}}\,a_{k\,{\rm a}}$ with their expectation values, $N_{\rm h}\,\delta_{j\,k}/(2S+1)$ and $N_{\rm p}\,\delta_{j\,k}/(2S+1)$, respectively, with $N_{\rm p}$ ($N_{\rm h}$) being the number of particles (holes). Thus we can treat the operators $\{C_{LM}\}$ as bosons when $N_{\rm p}$ and $N_{\rm h}$ are small compared with the number of the single-particle states ($= 2S+1$ on a sphere).

The pair creation/annihilation terms in the Hamiltonian are important in determining the electron correlation in the ground state, while the HF approximation neglects these terms. Now we fully take account of these terms in the boson approximation.

A key finding here is that the diagonalization is done with a unitary transformation, originally introduced by Bogoliubov, *i.e.*,

$$\mathcal{H}_{\rm eff} = \sum_{L,M} \omega_L\, D^\dagger_{LM}\, D_{LM} + E_0, \qquad (5)$$

$$\omega_L \equiv \sqrt{e_L\,(e_L + 2\lambda_L)}, \qquad (6)$$

$$D_{LM} \equiv U_{\rm S}\, C_{LM}\, U_{\rm S}^{-1} = C_{LM}\, \cosh(\theta_L/2) + C^\dagger_{L,-M}\, \sinh(\theta_L/2), \qquad (7)$$

$$U_{\rm S} \equiv \exp\left[-\frac{1}{4} \sum_{L,M} \theta_L \times \left(C^\dagger_{LM}\, C^\dagger_{L,-M} - C_{LM}\, C_{L,-M} \right) \right]. \qquad (8)$$

Here the crucial transformation $U_{\rm S}$ is a "pseudo-rotation" operation belonging to the Lie group SU(1,1) [11], and this indeed diagonalizes the Hamiltonian with the angle of pseudorotation $\theta_L = \coth^{-1}(1 + e_L/\lambda_L)$ as far as we regard $\{C_{LM}\}$ as bosons.

The ground state is then neatly expressed as

$$|\Psi_0\rangle \;=\; U_{\rm S}\,|\Psi_{\rm sym}\rangle, \qquad (9)$$

which involves a series of repeated creation/annihilation of exciton pairs if we expand the exponential form in eq.(8). We can call the state the vacuum of the transformed boson, $\{D_{LM}\}$, with $D_{LM}\,|\Psi_0\rangle = U_{\rm S}\,C_{LM}\,|\Psi_{\rm sym}\rangle = 0$. The energy E_0 appearing in eq.(5) is the eigenenergy of Ψ_0. In the language of the quantum optics, $U_{\rm S}$ precisely corresponds to a "squeezing operator" [12] so that we can call the ground state a squeezed vacuum state of the original boson $\{C_{LM}\}$.

Since the boson considered here is related to pseudospins via the relation, $s_y \propto C^\dagger_{00} - C_{00}$ and $s_z \equiv \sum_j (\, a^\dagger_{j\,\uparrow}\, a_{j\,\uparrow} - a^\dagger_{j\,\downarrow}\, a_{j\,\downarrow}\,)/2 \propto C^\dagger_{00} + C_{00}$, where $a^\dagger_{j\,\sigma}$ creates an electron in a j-th Landau orbit with pseudospin (*i.e.*, layer index) σ, the boson-squeezing can be recast into a pseudospin-squeezing. For this purpose it is convenient to rotate the pseudospin axes around the y axis by -90 degrees ($s'_x = s_z$, $s'_y = s_y$, $s'_z = -s_x$), where $s'_z = \sum_j (\, a^\dagger_{j\,{\rm a}}\, a_{j\,{\rm a}} - a^\dagger_{j\,{\rm s}}\, a_{j\,{\rm s}}\,)/2$ is now half the difference in the numbers of electrons between the antisymmetric and symmetric states. Then we have

$$|\Psi_{\rm sym}\rangle = |\, s, s'_z = -s\,\rangle, \qquad (10)$$

$$U_{\rm S} = U'_{\rm S}\; U(-\theta_0/2), \qquad (11)$$

$$U(\theta) \equiv \exp\left\{ \frac{\theta}{4s}\,[\,(s'_+)^2 - (s'_-)^2\,] \right\} = \exp\left\{ \frac{\theta}{2}\left[(C^\dagger_{00})^2 - (C_{00})^2 \right] \right\}. \qquad (12)$$

Here s is half the number of electrons, and the squeezing operator $U_{\rm S}$ has been decomposed into the $(L, M) \neq$

$(0,0)$ component, $U_{\rm S}^{'}$, and the $L = M = 0$ component, $U(-\theta_0/2)$, where $\theta_0 = \coth^{-1}(1 + \Delta_{\rm SAS}/\lambda_0)$ with $\lambda_0 = \sum_J^{'}(2J+1)\,(V_J^{\uparrow\uparrow} - V_J^{\uparrow\downarrow})/(2S+1) > 0$. The above expression, written in terms of $s_+^{'} = (s_-^{'})^\dagger = \sum_j a_{j\,{\rm a}}^\dagger\, a_{j\,{\rm s}}$ (the raising/lowering operators for pseudospin) enables us to regard that the operator $U(\theta)$ exactly corresponds to the "spin-squeezing" operator [13].

The starting state $\Psi_{\rm sym}$ has a mean pseudospin oriented along the s_x axis with circular variances of $(\delta s_y)^2 = (\delta s_z)^2 = s/2$. Thus, $\Psi_{\rm sym}$ is a spin-coherent state [14], which is defined as a state satisfying the minimum uncertainty relationship with variances $s/2$ equally distributed over the two orthogonal components normal to the mean pseudospin vector $\langle\, \mathbf{s}\, \rangle$ (Fig.1a). As the ground state evolves from $\Psi_{\rm sym}$, a spin-squeezing operator $U(\theta)$ compresses the fluctuations of the pseudospin in one direction at the expense of enhanced ones in the other direction [13,15].

We can rewrite the part, $U(-\theta_0/2)\,|\, s, s_z^{'} = -s\,\rangle$, in a more manifestly spin-squeezed form in terms of the spin-raising operator only [15] as

$$|\,\zeta\,\rangle \equiv U(\tanh^{-1}\zeta)\,|\, s, s_z^{'} = -s\,\rangle \tag{13}$$

$$= (\,1 - |\zeta|^2\,)^{1/4}\, \exp\left[\frac{\zeta}{4s}\,(s_+^{'})^2\right] |\, s, s_z^{'} = -s\,\rangle, \tag{14}$$

where the Campbell-Baker-Hausdorf formula is used with $\zeta = -\tanh(\theta_0/2)$. Then the ground state can be expressed as

$$|\Psi_0\rangle = U_{\rm S}^{'}\; |\,\zeta\,\rangle. \tag{15}$$

From this, we can confirm that the spin-squeezed state $|\,\zeta\,\rangle$ with a squeezing parameter of $\zeta = -\tanh(\theta_0/2) < 0$ has a squeezed variance $(\delta s_z)^2 \simeq (s/2)\, e^{-\theta_0}$ and an enhanced one $(\delta s_y)^2 \simeq (s/2)\, e^{\theta_0}$ for large s with fixed $\langle s_z \rangle = \langle s_y \rangle = 0$. Since s_y and s_z commute with $U_{\rm S}^{'}$ within the boson approximation, the ground state Ψ_0 has the same fluctuations, δs_z and δs_y, as a spin-squeezed state $|\,\zeta\,\rangle$ (Fig.1b), while the mean pseudospin is still oriented along the s_x axis for $\Delta_{\rm SAS} > 0$.

Let us now come to the physical interpretation of the pseudospin-squeezing. In the starting state $\Psi_{\rm sym}$, pseudospin 1/2 of each of the N electrons is oriented to the s_x direction in an uncorrelated manner. In fact, the variance $s/2 = N/4$ of each of the two components normal to the mean pseudospin is simply the sum of variances of the individual pseudospins each having the variance of 1/4 [13]. On the other hand, a spin-squeezing $U(-\theta_0/2)$ with $\theta_0 = \coth^{-1}(1 + \Delta_{\rm SAS}/\lambda_0)$ results from the electron correlation (*i.e.*, exchange interactions and charging energies) as well as from pair creation/annihilation of excitons across the gap $\Delta_{\rm SAS}$. These (nonlinear interactions in the quantum optical language) give rise to the squeezing of the total pseudospin. When the squeezing $U(-\theta_0/2)$ reduces δs_z and enhances δs_y, the fluctuation of the azimuth angle $\delta\varphi$ of the total pseudospin (the variable conjugate to s_z) is enhanced, so that φ tends to be ill-defined as the squeezing becomes stronger.

We have seen that the degree of squeezing can be controlled by varying the two parameters, d (which determines λ_0) and $\Delta_{\rm SAS}$. A strongly spin-squeezed state may be obtained for a sample with small $\Delta_{\rm SAS}$, which is easier to attain in bilayer hole gases. The boson approximation becomes worse as $\Delta_{\rm SAS}$ is decreased, but it has been shown that the boson approximation gives quantitative energetics (such as the pseudospin-wave dispersion that is either gapless or gapful) even for small $\Delta_{\rm SAS}$ [16]. We expect that the wave function is qualitatively given by the approximation there.

If a phonon mode couples with the strongly squeezed electron state, phonons may be concomitantly squeezed. A phonon squeezing in a three-dimensional solid driven by light pulses has recently been observed [17], but we may expect a phonon squeezing in quite a different context here. Thus an interplay among the excitons, phonons, and irradiated photons is an intriguing future problem.

Now we turn to the possibility of the exciton condensation in the bilayer QH system discussed in the literature [4,10]. In the special case of $\Delta_{\rm SAS} = 0$, the pseudospin-wave excitation spectrum does resemble the Landau spectrum in a superfluid ^{4}He, *i.e.*, having a gapless phonon-like mode and roton-like minimum. The exciton-condensed state may, in the present formalism, be captured as a "displaced squeezed state"

$$|\Psi_\alpha\rangle \equiv U_\alpha\, |\Psi_0\rangle, \tag{16}$$

$$U_\alpha \equiv \exp\left(\,\alpha\, C_{00}^\dagger - \alpha^*\, C_{00}\,\right). \tag{17}$$

Here U_α is a unitary displacement operator with a complex α [12], again in analogy with the quantum optics. This state has indeed a nonzero expectation value of an annihilation operator, $\langle\Psi_\alpha|\, C_{LM}\, |\Psi_\alpha\rangle = \alpha\, \delta_{L\,0}\, \delta_{M\,0}$, for a nonzero α.

With this unitary transformation the Hamiltonian reads

$$\mathcal{H}_{\rm eff} = \sum_{L,M} \omega_L\, D_{LM}^{\alpha\,\dagger}\, D_{LM}^{\alpha} + \omega_0 \left(\, \beta^*\, D_{00}^\alpha + \beta\, D_{00}^{\alpha\,\dagger} \right) + E_\alpha, \tag{18}$$

$$D_{LM}^\alpha \equiv U_\alpha\, D_{LM}\, U_\alpha^{-1} \;=\; D_{LM} - \beta\, \delta_{L\,0}\, \delta_{M\,0}, \tag{19}$$

where $\beta = \alpha\,\cosh(\theta_0/2) + \alpha^*\,\sinh(\theta_0/2)$. We can call Ψ_α the vacuum state of the boson $\{D_{LM}^\alpha\}$ since $D_{LM}^\alpha\, |\Psi_\alpha\rangle = U_\alpha\, U_{\rm S}\, C_{LM}\, |\Psi_{\rm sym}\rangle = 0$, so that it has an energy expectation,

$$E_\alpha \equiv \langle\Psi_\alpha|\, \mathcal{H}_{\rm eff}\, |\Psi_\alpha\rangle = E_0 + |\alpha|^2\, \left\{\, \Delta_{\rm SAS} + \lambda_0\, [\, 1 + \cos(2\phi)\,]\, \right\}, \tag{20}$$

where ϕ is the phase of α ($\alpha = |\alpha|\, e^{i\,\phi}$). For Ψ_α to be realized spontaneously, $E_\alpha \le E_0$ is needed, but eq.(20) has the opposite inequality since $\lambda_0 > 0$.

Then the only possibility for the exciton-condensed state to survive is when $E_\alpha = E_0$. This condition is satisfied only when $\Delta_{\rm SAS} = 0$ and α is pure imaginary. When $\Delta_{\rm SAS}$ is finite, the spontaneous symmetry breaking cannot be expected in fact due to a finite excitation gap ω_0. The second condition that α be pure imaginary implies that the displacement operator U_α amounts to a rotation about the z pseudospin axis, $\exp\left[\, i\,(2\,{\rm Im}\,\alpha/\sqrt{2S+1}\,)\, s_z\right]$, while $\beta = i\,({\rm Im}\,\alpha)\, e^{-\theta_0/2} \to 0$ and $\theta_0 \to +\infty$ for $\Delta_{\rm SAS} \to +0$. Thus the exciton-condensed state corresponds, in terms of the spin squeezing, to a *rotated-squeezed* state, $|\Psi(\varphi)\rangle = \exp(-i\,\varphi s_z)\, U_{\rm S}\,|\Psi_{\rm sym}\rangle$, reflecting the U(1) symmetry, while the BCS-like state, $\exp(-i\,\varphi s_z)\,|\Psi_{\rm sym}\rangle \propto \prod_j (a^\dagger_{j\uparrow} + a^\dagger_{j\downarrow}\, e^{i\varphi})|0\rangle$, discussed in Ref. [10] is a *spin-coherent* state that is compatible with the SU(2) symmetry. As seen, the exciton-condensed state is energetically allowed only in the absence of interlayer tunneling, where some stabilization mechanism will be further required to make the state the true ground state.

So far we have concentrated on $\nu = 1$, but we can construct the squeezed-vacuum wavefunction for fractional values of Landau level filling $\nu = 1/m$ ($m = 3, 5, \cdots$) via a composite-fermion transformation [18] as has been proposed for bilayer systems [19] by the present authors. In a spherical system this can readily be done, since everything is written in terms of angular momenta.

We are grateful to Shin Takagi, Tetsuo Ogawa, Satoru Okumura and Yasuo Okajima for illuminating discussions on the coherent state, and to Komajiro Niizeki for valuable comments. This work was in part supported by a Grant-in-Aid from the Ministry of Education, Science and Culture, Japan.

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FIG. 1. (a) A spin-coherent state (corresponding to $\Psi_{\rm sym}$) and (b) a spin-squeezed state (Ψ_0) are schematically shown, where the regions over which spins fluctuate are indicated by shading.

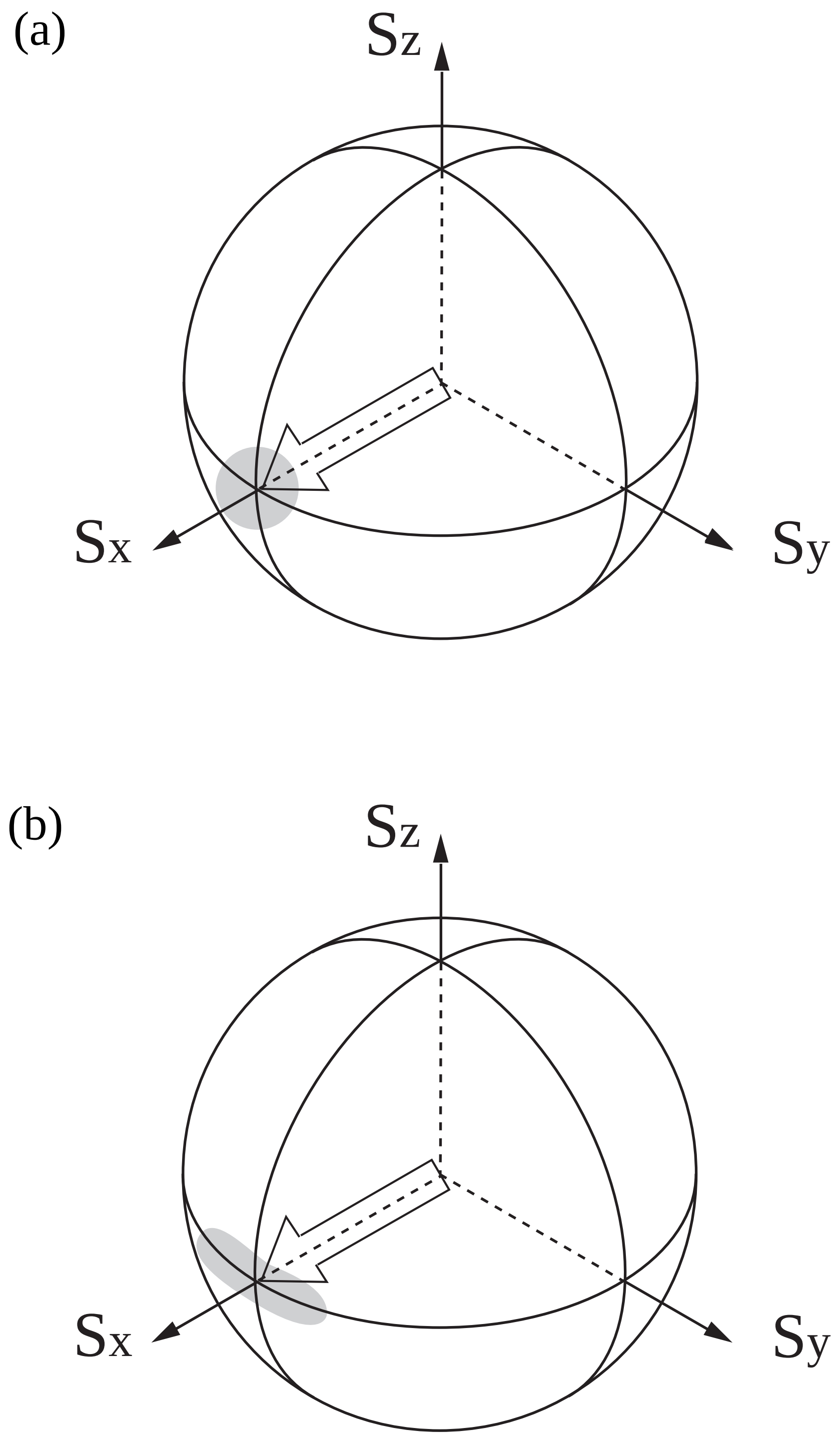

Fig.1. (Nakajima & Aoki)